

The Diffraction Grating

A diffraction grating is a special device that will cause multiple diffraction patterns which then overlap. A diffraction grating comprises an obstruction with a very large number of a sequence of equally-spaced narrow slits through which the waves can pass.

The diffraction grating is made by cutting lines on transparent plane glass plate by a sharp forehead of a piece of diamond. There are about 10,000 lines per centimeter. Width of each of the sheet is about 10^{-4} cm.

A plate having many fine slits of equal width placed side by side is called, diffraction grating.

Diffraction grating creates an interference pattern with a mathematically well defined spacing between bright and dark spots.

Grating constant

The distance from the starting of a slit to the starting of the next slit is called the grating constant. In other words it can be said that from the terminal end of a slit to the terminal end of the next slit is called the grating constant.

Explanation : Suppose the width of each slit of a grating = a
and width of each line = b

According to the definition, the grating constant, $d = a + b$

d is sometimes called grating element.

In the length ' d ' of the grating, number of line = 1

So, number of lines in unit length, $N = \frac{1}{d}$

$$\therefore N = \frac{1}{d} = \frac{1}{a + b}$$

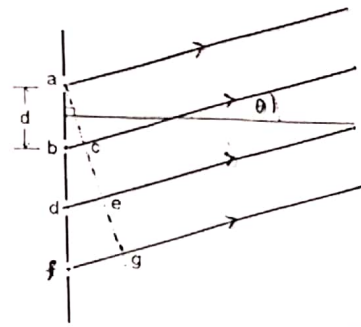
Two points between the interval $(a + b)$ of the grating is called corresponding points.

Uses of grating

Gratings are used for different purposes. Some of their uses are mentioned below :

- (1) To determine the wavelength of light.
 - (2) To separate two spectral lines of same wavelength.
 - (3) To determine the rate of change of diffracted angle with respect to the wavelengths.
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$$d \sin \theta = n \lambda$$



In the diagram right there will only be constructive interference at the screen if each wavelet arrives in phase and this can only happen if their path differences are whole numbers of wavelengths. So:

- bc has to be an integer, n , times the wavelength
- but $bc = d \sin \theta$ by trigonometry.

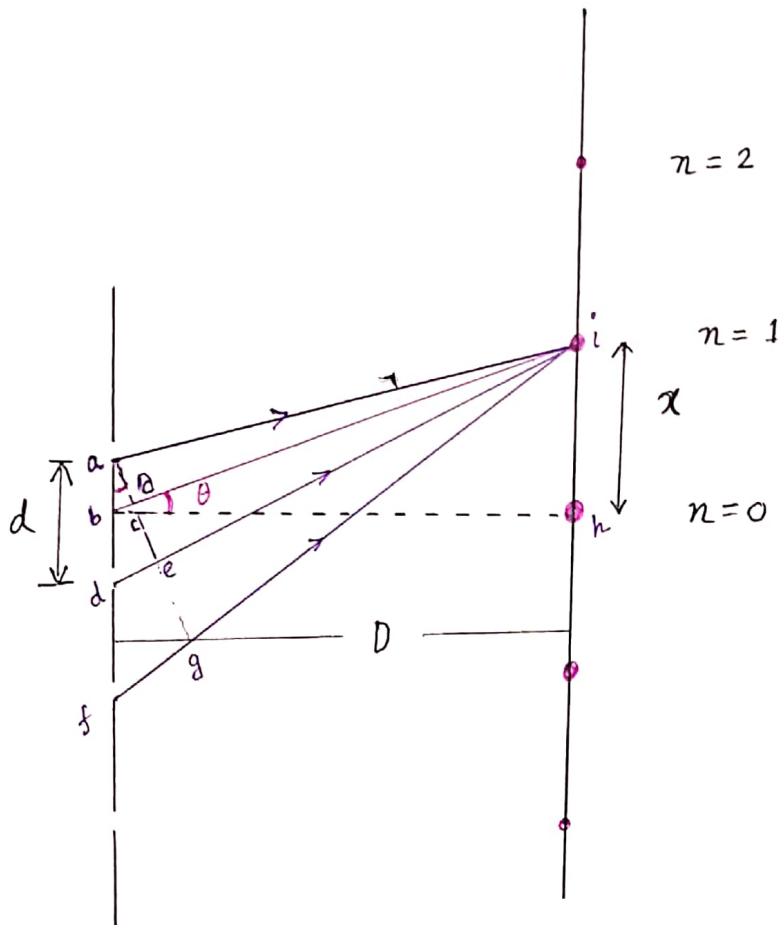
The pattern produced by each colour passing through a diffraction grating follows the equation:

$$n \lambda = d \sin \theta$$

where θ is the angle between the original direction of the waves and the direction of a bright spot. λ is the wavelength of the light used. d is the spacing between the slits on the grating, and n is called the 'order'. The order is the bright spot number from the central maximum (which is $n = 0$).

Students should be aware that n cannot increase indefinitely because the rays in the diagram always go forwards through the slits (i.e. $\theta < 90^\circ$). Experimentally they should be shown that the spacing of the spots on the wall depends on the line spacing in the grating and this affects the number of spots actually visible. They should therefore be taught to use the idea that $\sin \theta = \frac{n \lambda}{d} < 1$ and therefore $n < \frac{d}{\lambda}$.

Two common errors arise here: first, to fail to give the maximum order (n) as an integer (a result of $n < 4.88$ means $n = 4$ is the largest order visible). Second, to fail to recognise that this is the number of spots visible **on each side** of the central (zero order) spot and so the total number of **visible** spots is $2n + 1$.



At i , Path difference = $d_i - a_i = de = \lambda$
for constructive interference.

Through diffraction grating for multiple diffraction, path difference $de = n\lambda$

$$\text{In } \triangle ade, \quad \sin \theta = \frac{de}{ad} = \frac{n\lambda}{d}$$

$$\therefore d \sin \theta = n\lambda$$

$$\text{and in } \triangle bhi, \quad \tan \theta = \frac{hi}{bh} = \frac{x}{D}$$

If θ is very very small ($\lambda \ll d$)

$$\sin \theta = \tan \theta = \theta$$

$$\sin \theta = \tan \theta$$

$$\frac{n\lambda}{d} = \frac{x}{D}$$

$$n\lambda D = dx$$

$$x = \frac{n\lambda D}{d}$$

$x =$ fringe width .

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